Do Balanced Budget Rules Prevent Expenditure Smoothing? A Buffer-Stock Model of U.S. State Governments Says No.

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August 21, 2020

Abstract

This paper examines how state governments smooth expenditures in the U.S. over time in the face of cyclical fluctuations in revenue. Most states operate under balanced-budget constraints but state governments typically run surpluses some of which are deposited in "rainy day funds" which are designed to smooth expenditures. The magnitudes of these funds are too small to provide substantial expenditure smoothing; however, state governments hold unspecified cash and security balances, which dwarfs rainy day funds, are procyclical to states' revenue, and enable expenditure smoothing. We model state governments as Carroll (1997) buffer-stock consumers and estimate their implied preference parameters using indirect inference. The estimated behavioral parameters fit that of a rational consumer with moderate risk aversion and impatience.

1 Introduction

We study government expenditure patterns over the business cycle using data for 49 U.S. states government over the period 1975–2014. State governments expenditure in the U.S. amounts to about 10 percent of GDP and most states operate under balanced budget rules (Poterba, 1994), which are designed to prevent excessive state government debt, but may lead state governments to aggravate the impact of business cycles by contracting expenditures in recession as pointed out byRosengren (2018). The rationale for imposing balanced budget rules is that state governments are perceived to enjoy spending at same time as they are impatient and unwilling to accumulate enough savings unless constrained by rules. The buffer-stock model of Carroll (1997) is designed to capture the behavior of impatient consumers who face budget constraints and as this also describes state governments, we fit a buffer stock model to state level fiscal policy. We treat state governments as if they are an agent and do not model the more narrow preferences of politicians and their interactions with voters.

State governments (which we some times refer to as "states") have created explicit "rainy-day funds" to be drawn upon in recessions.[1] Zhao (2016) evaluates statistically whether rainy day funds are large enough to absorb typical fluctuations in state taxes collected and find that many states appear to save too little in their rainy days funds. We compare rainy day fund balances to variation in revenue and it seems clear that rainy day fund balances are

[1]About half of U.S. states have instituted such funds over the last forty years and now all states, save Montana, have such funds.

too small to accomplish meaningful consumption smoothing. We show in this paper that substantial consumption smoothing by states is accomplished by running down cash balances in recessions and replenishing them in booms. We are not aware of any evidence that general cash balances are directly motivated by a desire for expenditure smoothing and the role of states' cash balances for shock absorption seems not to have been previously studied in the academic literature.

A larger body of work has modeled governments as consumers using the permanent income hypothesis (PIH) of Hall (1978). Such work takes the government to be an agent who receives exogenous "labor income" (income excluding interest on assets) and derives utility from "government consumption," typically identified with government expenditure. The literature has tested the strong prediction of the PIH model that consumption is a random walk and rejected it. Campbell and Mankiw (1989) suggest an extension of the PIH model, labelled the "rule-of-thumb" consumer model, where a certain fraction of agents consume their current income (the rule-of-thumb consumers) while the remaining fraction behave as prescribed by the PIH. This model is ad hoc, but provides a useful measure of deviations from the PIH. **?** estimate the rule-of-thumb consumer model for aggregate U.S. state and local spending and find that state and local government spending follows available resources, in other words, the average state/local government is a rule-of-thumb consumer. However, the use of aggregated data may hide important variation across the many different state and local governments. Using disaggregated data for state governments, a number of papers have rejected the PIH model and some have extended the model to include rule-of-thumb consumers; see, e.g., **?**. **?** estimate the rule-of-thumb model for Norwegian municipalities and find important variation which allows them to examine if entities that face tougher fiscal environments are more likely to display rule-of-thumb behavior—throwing light on whether such behavior is a reflection of constraints rather than behavioral myopia.[2] Biolsi and Kim (2019) estimate a second-order approximation to the Euler equation and find significant effects of balanced-budget rules.

Work in the Dynamic Stochastic General Equilibrium (DSGE) modeling tradition has produced quantitative insights for optimal government debt. For example, **?** calibrate a closed economy model to post World War II U.S. data, where government debt helps atomic agents smooth consumption over time at the cost of adverse incentive effects and find that it predicts a level of government debt around 60 percent of GDP. This modeling choice is also adopted by the recent sovereign debt literature which typically assumes a benevolent government which borrows on behalf of consumers. This literature is way too large to survey here, but early papers like **?** assumed extremely large discount rates for governments in order to fit the data although recent work such as **?** or **?** assume discount rates in the range of 5 percent annually, which is only slightly larger than typical consumer discount rates.

The buffer-stock model departs from the vantage point of the PIH model in assuming forward looking agents who take income as exogenous, but it relaxes the assumptions of quadratic utility and lack of credit constraints underlying the PIH martingale result. As the

[2]**?** shows that, if government expenditures are exogenous and tax collection costs are increasing in tax rates, governments should (if they are efficient) smooth taxes over time and—for typical modeling choices—tax rates should behave like random walks. The Barro tax smoothing model successfully explains why national governments run deficits in the face of large spending shocks, such as wars and devastating earthquakes, but it has met with little success in explaining more normal fluctuations in government deficits and saving.

? model, the buffer-stock model predicts a target level of government saving, which the PIH and Barro models do not. In the Carroll (1997) model, the trade-off between prudence (in **?**'s sense) and impatience combined with a probability of zero income leads state governments to have target level for savings, often referred to as a buffer stock of savings. We estimate the model using indirect inference, matching two informative statistics from the data to corresponding statistic from model simulations, namely, 1) governments stock of savings to the predicted target level generated from the model and 2) the propensity to consume (expend funds) as a function of the deviation of cash holdings from the target level. As we fit both spending and savings levels, our approach can be seen as combining features of the literature considering optimal saving and the literature considering optimal expenditure smoothing.

We fit U.S. state government expenditures, revenues, and savings to the buffer-stock model using a panel data set for U.S. states over the period 1976–2014. We follow Craig et al. (2016), who modeled the U.S. unemployment insurance system, using the framework introduced by Jappelli, Padula and Pistaferri (2008) (hereafter JPP), building on the model of Carroll (1992, 1997). The model is particularly well suited for modeling fiscal policy under the constraints imposed by balanced budget rules,because the model explicitly allows for credit constraints.[3] Clearly the perspective underlying our work is derived from an "as if" description of preference irrespective of the objectives of any of the myriad individual actors within the state government. This may or may not be useful, but the proof is in

[3] Caselli and Reynaud (2014) study whether fiscal rules affect fiscal outcomes and find, using instrumental-variables estimation, that well-designed rule do, confirming that fiscal outcomes often are the shaped affected by external constraints.

the pudding; i.e., do we get significant results with a clear interpretation? We do, and our results characterize the policy outcomes of the government as resulting from an aggregate utility function consistent with rational consumer behavior. We avoid the entire subject of public choice and the principal-agent relations between voters and governments and focus on the outcomes to assess whether state government expenditure patterns are consistent with those of a rational buffer-stock consumer.

A challenge for fitting government behavior to a consumer model with exogenous income is that governments set tax-levels as well as expenditure levels. Our approach is to consider the part of tax fluctuations that follows (state-level) personal income fluctuations as exogenous, while we consider tax-breaks (hikes) relative to this benchmark as a deduction from consumption. The consumption, as we model it, is government current expenditure which includes "government consumption," as defined in the national accounts, transfers to individuals and local governments, as well as the tax-breaks mentioned.

A second challenge in fitting U.S. state government behavior is that constraints are imposed by the federal government. In particular, medicaid is an important component of state government expenditure with a large fraction reimbursed by the federal government. While state have some discretion over Medicaid spending as state can opt in and out of certain features (see Craig and Howard (2014)), the size of the program and extensive federal intervention suggests that year-by-year spending does not solely reflect the choices of state officials. We therefore estimate our model on assuming Medicaid expenditure is exogenous and separable from other expenditures and the state's share of medicaid taxes is predetermined and not fungible with other revenue. I.e., we estimate the model on revenue and expenditure data with Medicaid spending subtracted from both.[4]

We find that U.S. states are well modeled as agents with moderate risk aversion and discount rates, although governments are still impatient in the sense that their discount rate exceeds the interest rates—a condition for a target buffer stock to exist.[5] This result is driven by two features of the data. First, state governments on average hold large cash balances—much larger than the rainy day funds that are ostensibly for this purpose. These funds are not aggregated in a separate savings account, but appear to be scattered across the state accounts, which may be why they have not attracted attention in the literature, but we show that the cash balances are strongly pro-cyclical.[6] Second, state government consumption is found to react less than one-to-one to increases in revenue.

It is reasonable to expect that preferences between state governments might vary. Our pooled estimates can be interpreted as average values and we do not have enough degrees of freedom to explore heterogeneity between state governments in depth, but it is well known from, e.g., Poterba (1994) that the stringency of balanced budget rules varies by state. To see how this affects the parameters estimates of the model, we group the states into stricter and laxer balanced budget rules. We can think of two obvious reasons for why estimated patience may be different for states with stricter balanced budget rules: a) states behave like

[4]In the online appendix, we display results generated under the assumption that medicaid spending is fully substitutable with other spending.

[5]The exact condition is a little more complicated, depending on average growth and the variability of permanent income shocks. We will state it in the model section.

[6]For example, New York's cash balances under cash and securities holdings seem to include SUNY funds, although there is no direct evidence that state smooths consumption on the back of its universities. In Texas, 50% of any state surplus is required to be deposited in the rainy day fund. The regulations appear silent, however, on the other half.

patient consumers because stringent rules force them to do so or b) states that typically have impatient politicians need rules to constrain themselves.[7] We find that state governments facing stricter budget rules behave less patiently, consistent with the latter reason.

The rest of the paper is organized as follows. Section 2 describes the data and the patterns of government income and expenditure over the period 1975–2014. This includes a discussion of the cash reserves in state government accounts. Section 3 presents the buffer stock model and describes how we adjust taxes and expenditure to correspond to "exogenous income" and "consumption" in the model. Section 4 presents estimates of the buffer stock model, and then fimd the estimates of the preference parameters using Indirect Inference. Section 5 presents the results and Section 6 concludes and briefly considers the implications of the estimates for more explicit institutions to provide a savings function for consumption smoothing.

2 Data

We construct a panel of 49 U.S. states (all, except Alaska) from 1975 to 2014.[8] Data on State Government finances is obtained from the U.S. Census Bureau's Annual Survey of State and Local Government Finances. For state government revenue, we use taxes from all sources while for state government expenditures, we use total current expenditures, excluding capital

[7]The earliest constraints were imposed by states that had defaulted on debt due to excessive canal building in the mid-1800s.

[8]We stop in 2014 because the data revisions for 2016 seem to be incomplete. We drop Alaska as our estimates were sensitive to our results based on a leverage test, see Young (2019). Alaska is clearly an outlier state with per capita cash balances 400% bigger than the other 49 states due to the large oil funds that the state maintains.

spending (capital spending is often financed by earmarked debt, which we are not analyzing). Typically, states adjust expenditure slowly, see Sørensen, Wu and Yosha (2001), and some states set their budget biannually, so we treat two non-overlapping years as one time period for the remainder of the paper. We take two-year sums for flow variables such as revenue, expenditure, etc. and use the end-of-year value for the year before the two-year period for stock variables such as cash and security holdings etc. The first period in our sample is 1975–1976 and, to illustrate, for this period the flow variables are the sum of the 1975 and 1976 numbers and the stock variables are at the end of 1974.

Figure 1 shows that states on average run budget surpluses. If these are retained as savings, this will provide buffers that can used to smooth expenditures in the face of fluctuations in revenue. In good times, state governments can, and often are required to, save in the rainy day fund but we show that states also build up other cash balances. While the former has been discussed in the literature, the role of the later is unexplored. We define "cash" as the balance of the cash and securities account, after subtracting balances earmarked for pensions, insurance and bond funds, and interest payments on debts. We show that these cash balances are very pro-cyclical and, although the census does not classify these funds one way or another, we find that they to a first approximation can be treated as unrestricted savings.[9] Figure 2 shows fluctuations in cash holdings relative to expenditures, taxes, and rainy day fund balances for the period 1975–2014. The gray bars illustrate recession periods based on the NBER definition. Cash balances are cyclical, partly off-setting the fluctuations

[9] It is likely that "cash," as we define it are subject to various constraints which differ by state and it is also the case that states can delay pension fund contributions; see for instance www.pewtrusts.org/en/research-and-analysis/issue-briefs/2019/06/the-state-pension-funding-gap-2017.

in revenue. Current expenditures are pro-cyclical with a very long lag, declining for years after the end of recessions. Rainy day funds are pro-cyclical but it is hard to see in the figure because the magnitudes of these funds are tiny compared to fluctuation in taxes—clearly, the rainy day funds at present absorb only a small fraction of shocks. Figure 2 shows that cash balances at the end of the fiscal year fall in three recessions while in the 2000 dot-com recession the growth rate plunges dramatically. This illustrates that cash serves to cushion down periods. Some budget documents support this claim: for instance New York's document explicitly mentions that these cash balances are used for smoothing revenue shortages.

Another way to look at the extent to which states are smoothing their expenditure is to look at the standard deviation of expenditure and revenue, see Table 1. We find that standard deviations (both cross sectional and time series) of expenditures is smaller than those of revenues implying that there are smoothing mechanisms which states are using to absorb some of the revenue fluctuations. The data in the table further support that intergovernmental revenue does not serve to smooth state government consumption, as we see that the standard deviation for own revenue is smaller than that for total revenue. Because the difference in the two measures is inter-governmental receipts, this implies that intergovernmental transfers are increasing the variance as own revenue is total revenue net of intergovernmental transfers.

Based on these observations we hypothesize that states smooth current expenditure through cash balances and we construct a model that we will calibrate to average savings and propensities to use cash-on-hand for expenditures.

3 Model

We assume that each state government acts as a single entity behaving like a buffer stock consumer. Specifically, state governments hold savings in order to not have to curtail spending dramatically in cyclical downturns at the same time as they are impatient in that they would rather expend resources now rather than later.[10] The buffer stock model allows these forces to be off-setting to the extent that agents do not build up savings beyond a certain level while holding enough saving that consumption can never go to zero. Previous studies such as Craig et al. (2016) have shown that for the independently operated unemployment insurance funds, states exhibit behavior similar to that of a buffer stock consumer. That is, governments are sufficiently risk averse to save, yet sufficiently impatient to spend resources as soon as they exceed a target level. Based on the degree of risk aversion and impatience and the underlying risk to revenue, government will hold smaller or larger buffer stocks of cash.

We describe the buffer stock model as formulated by Carroll (1992) where agents face permanent and transitory shocks and cannot borrow. Furthermore, there is a small probability of zero which guarantees that households always holds a positive amount of savings. (Carroll (1997), Zeldes (1989), Schechtman (1976)).

[10]Impatience may be a function of government structure or reflecting the impatience of voters.

3.1 The Buffer Stock Model

We assume the economy is populated by infinitely lived states which maximize the present discounted value of utility derived from consumption.[11] The objective of the state is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\rho}}{1-\rho} \tag{1}$$

where β is the time discount factor, $C_t > 0$ is consumption, and $\rho > 0$ is the coefficient of relative risk aversion. The dynamic budget constraint is

$$W_{t+1} = (1+r)(W_t + Y_t - C_t) \ , \tag{2}$$

where W_t is financial wealth, r is the interest rate, and Y_t is income. Following Carroll (1992), we define cash-on-hand ("COH") as the beginning of period savings plus current period income, $COH_t = W_t + Y_t$.

Income is modeled as the product of a random walk permanent component and an i.i.d. transitory component:

$$Y_t = P_t \epsilon_t \tag{3}$$

$$P_t = G P_{t-1} \psi_t \ , \tag{4}$$

[11] Because we are concerned with state governments and not politicians, we assume that state governments cannot die and we are agnostic about changes in political ideology or regime.

where P_t is the permanent component of income, G is a deterministic growth rate of income, ϵ_t is a transitory shock to income and ψ_t is a shock to permanent income. We extract these components using a kalman filter approach and will discuss that in the following section. The distribution of the permanent and transitory shocks to income are

$$log(\psi_t) \sim N(-0.5\sigma_\psi^2, \sigma_\psi^2)$$

$$\epsilon_t = \begin{cases} 0 & \text{with probability } \omega \\ \theta/(1-\omega) & \text{with probability } 1-\omega, \text{ where } log(\theta_t) \sim N(-0.5\sigma_\theta^2, \sigma_\theta^2) \end{cases} .$$

We normalize all variables by permanent income, making them stationary.[12] Normalized variables are in lower case: $c_t = C_t/P_t$, $coh_t = (W_t + Y_t)/P_t$, and $y_t = Y_t/P_t$. States are not allowed to borrow which corresponds to balanced budget rules in the data. The key parameters are the discount factor and the risk aversion coefficient. The discount factor measures the patience of the state government while risk aversion measures how averse are state governments to variation in expenditures (across states-of-the-world and over time).

3.2 Mapping of Variables

Since the consumer model we use to describe government behavior is defined in terms of consumption and exogenous income, it takes some re-interpretation to map government expenditure and revenue into these concepts.

[12] Because of homotheticity of the utility function, the analysis can be accomplished using the normalized variables and this allows us to eliminate permanent income as a state variable; see Carroll (1997).

3.2.1 Income

State government revenue consists of taxes on both individuals and firms and transfers from the federal government—for brevity, we will refer to "taxes."[13] When we add transfers from the federal government, we net out medicaid expenditures because it comprises a major part of the transfers.[14] To define "income" of state governments to match up with the exogenous income in the model, we assume that changes in revenue due to changes in state-level personal income is exogenous to state government spending. While this is an approximation, we believe reverse causality at the state level is a minor issue, especially compared to the consumer case where individuals may take on, say, a second job in order to achieve a consumption target. We estimate this exogenous components by regressing taxes on personal income, allowing for time fixed effects:

$$\Delta Taxes_{it} = \gamma_t + \gamma \Delta PersonalIncome_{it} + \epsilon_{it}, \tag{5}$$

where $PersonalIncome_{it}$ is the aggregate personal income of residents in state i at period t. We estimate the regression in growth rates because both tax revenue and personal income are highly persistent and we cannot reject unit roots.[15] State income, $\widehat{Taxes}_{it}$, is the fitted values of this regression in levels, which we obtain by defining $\widehat{Taxes}_{i1} = Taxes_{i1}$ and recursively

[13]We exclude enterprise funds (such as utilities and airports) from both revenue and consumption.

[14]In our robustness checks, we include medicaid in the intergovernmental transfers and find that the estimated behavioural parameters indicate that state governments still exhibit a buffer stock behavior and are much more patient.

[15]Estimating this in levels will result in a spurious regression with non-standard distributions of the t-statistics.

computing

$$Y_{it} \equiv \widehat{Taxes}_{it} = \Delta\widetilde{Taxes}_{it} + \sum_{j=2}^{t} \widehat{Taxes}_{ij-1} \ , \tag{6}$$

where $\Delta\widetilde{Taxes}_{it}$ is the fitted value of the regression in differences.[16] Thus equation 6 gives the exogenous income of the state.

3.2.2 Extracting permanent and transitory income using Kalman filter.

In order to extract the permanent and transitory component of state's income, we implement a Kalman filter to estimate these unobservables. For that we must reformulate the statistical model of the income process in state space form. This includes a transition equation which describes the evolution process of the the state variable and a measurement equation which describes how the latent state variables affect the observed variables each period. The general form of the transition equation is

$$\alpha_t = K\alpha_{t-1} + R\eta_t \ , \tag{7}$$

where K and R are matrices of constants and $\eta_t \sim N(0, Q)$. While the general form of the measurement equation is

$$m_t = Z\alpha_t \ , \tag{8}$$

[16]We treat two years as one period and take non-overlapping sums (for flow variables such as income). The first period is 1971 to 1972. Once we obtain the whole series, we eliminate $\widehat{Taxes}_{i1}$ from the series because this is the initial value obtained from the data and not a residual.

where Z is again a constant matrix. For our analysis we will be estimating the income process which has a permanent component which follows a random walk as well as the transitory component which follows an i.i.d process. The log of the income process is

$$log(Y_t) = log(P_t) + u_t \tag{9}$$

$$log(P_t) = log(P_{t-1}) + e_t \ , \tag{10}$$

where $u_t = log(\epsilon_t)$ and $e_t = log(\psi_t)$. We take equation (9) as the measurement equation and equation (10) as the transition equation. The state space representation of this income process consists of states

$$\alpha_t = \begin{pmatrix} e_t \\ u_t - u_{t-1} \\ -u_t \end{pmatrix} \qquad \eta_t = \begin{pmatrix} e_t \\ u_t \end{pmatrix},$$

the restriction matrices are

$$K = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}$$

The transition equation is then

$$\begin{pmatrix} e_t \\ u_t - u_{t-1} \\ -u_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_{t-1} \\ u_{t-1} - u_{t-2} \\ -u_{t-1} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} e_t \\ u_t \end{pmatrix}$$

and the covariance matrix of the error terms is

$$Q = \begin{pmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_t^2 \end{pmatrix}$$

where σ_ψ^2 is the variance of permanent shocks and σ_θ^2 is the variance of transitory shocks. [17] Kalman smoother estimates the unobserved states by backward forecasting using the kalman filter estimates of the parameters and thereby extract the permanent and transitory components of state's income. We normalize the variables in our analysis with the permanent income estimates that we thus obtain. Kalman filter methodology that we employ is an improvement to the Friedman's definition of permanent income which used 6 year moving averages as it imposes less restrictions on the underlying structure of the permanent and the transitory components of the income.

[17]Given the state space representation of the statistical model, the Kalman filter proceeds by predicting the current state and conditional covariance matrix of the prediction errors based on past states, computing the loglikelihood function conditional on past observations then updates the states given current and past observations and the conditional covariance matrix of the states given the updated states. We perform the estimation in two steps. In the first step, we initialize states at zero. In the second step, we initialize at the stationary distribution.

3.2.3 Consumption

We define state government consumption ("consumption") as current expenditure minus the part of taxes that are due to rate changes, rather than changes in state personal income. Note that this concept of consumption is very different from government consumption in the national accounts, which does not include transfers. The idea behind this is that policy-makers receive political benefits from spending that increase the utility of the representative consumer in their state and from discretionary declines in taxes.[18] We also net out medicaid from our definition of consumption as it contains transfers from the federal government and hence will not capture pure state government's behaviour.

We defined discretionary taxes as the residual

$$\widehat{u_{it}} = Taxes_{it} - \widehat{Taxes}_{it}, \tag{11}$$

and consumption as

$$C_{it} \equiv CurrentExpenditure_{it} - \widehat{u_{it}} \ . \tag{12}$$

3.2.4 Wealth and Cash-on-Hand

We define state financial wealth ("W") as cash balances in the cash in securities account. These do not include cash earmarked for offsets to debt, bond funds, insurance trust funds

[18] We do not include capital outlays because they are primarily financed by debt.

or pension funds. [19] We define cash-on-hand as

$$COH_{it} = W_{it} + Y_{it} \ .$$

Cash-on-hand is the beginning of period resources available for consumption.[20]

4 Empirical Strategy

We estimate the model in two major steps. First, we estimate the propensity to consume out of cash-on-hand, denoted θ, which was suggested as an informative statistic by Jappelli, Padula and Pistaferri (2008) (JPP). For our SGMM estimation, it is not essential to use this statistic but it allows us to compare to their numbers for consumers. Further, we calculate average cash-on-hand (relative to permanent income). Second, we estimate the two preference parameters of the buffer stock model, the discount rate β and the degree of risk aversion ρ by simulating the model for a large number of artificial state governments, calculating θ and average cash-on-hand and finding the values of the parameters that bring these moments closest to the corresponding data values.

[19] In the budget documents they are called other funds under cash and security accounts.

[20] Rainy Day Funds make up a small fraction of cash and securities holdings, even after we have subtracted pension and insurance trust assets. Our data set only goes back to 1988 but Zhao (2016) finds that some states have had similar funds that go back farther than this. Upon examining state legislation, we also find that these funds have existed since the 60's and maintain purposes similar to Rainy Day Funds but are occasionally renamed.

4.1 JPP Regression

The buffer stock model implies that state governments behave as if they have a target level of wealth. That is, the state government desires to have sufficient savings on hand to be able to weather an unexpected shock to income and also desires to spend as much money in the current period as possible consistent with the desire for savings. We derive the target level of wealth, or target cash-on-hand (coh), as the average of coh across states and years in the sample. Using this definition of target wealth, we estimate the statistic innovated by Jappelli, Padula and Pistaferri (2008) which they call the covariance ratio. This is defined as

$$\theta = \frac{Cov(coh_{it} - coh^*, c_{it})}{Cov(coh_{it} - coh^*, coh_{it})} \ . \tag{13}$$

The covariance ratio is identical to an Instrument Variable (IV) estimate of consumption on cash-on-hand, using the deviation of cash-on-hand from the target level as the instrument. Cash-on-hand is predetermined and this would make θ an unbiased estimate of the propensity to consume out of cash-on-hand under ideal circumstances (dynamics in the relations would endanger the causal interpretation) so we will consider the estimate suggestive but not necessarily unbiased. Our main goal is not to estimate θ but the deep parameters of the utility function. JPP show that this satisfies the theoretical requirement that

$$\theta > 1 - \frac{G}{(1+r)e^{\sigma_\psi^2}} \ , \tag{14}$$

with the interpretation that if the deviation from target cash-on-hand, $coh_{it} - coh_i^*$, is positive then impatience will dominate risk aversion. When impatience is dominant then state governments will increase consumption and run down their savings. Conversely, if current savings falls short of the target wealth, risk aversion will dominate. In this case, state governments will desire to decrease their current consumption to accumulate savings. We estimate θ using standard IV and obtain standard errors that are informative about the precision of the SGMM estimates.

4.2 Discussion of Results

Table 4 displays the means of the model variables as well as the average standard deviations across time and states. The first line of the table shows that the overall average of cash-on-hand is 1.25 times that of permanent income. The average standard deviation of cash-on-hand over time for the cash-on-hand is not as large as the average standard deviation across states, which implies that states do not wish to deviate too far from their target savings. The table also shows average political income, and average political consumption.

Table 6 shows the result of estimating θ. Presence of balance budget rules have been thought to restrict state's ability to do inter-temporal smoothing. Hence we estimate θ for the full sample and then split the sample into states with strict balanced budget rules (BBR) and loose balanced budget rule. We use an index which captures the strictness of BBR's to divide the sample into 25-24 states. This index is constructed by ACIR (1987) and has been used by studies on balanced budget rules **??**. In column (1), we provide estimates for the pooled sample while columns (2) and (3) show estimates for states with loose and tight balanced budget rules. For the pooled sample we find a propensity to consume 0.56 which is significantly different from zero at the 5% level.[21] The interpretation of θ is that when the realized cash-on-hand is below (above) target, the government will decrease (increase) consumption and save (dis-save) until they reach their target. So the estimates indicate that when states income falls by a dollar, such that they move below their target wealth, states reduce their consumption by 0.56 cents. In column (2), we see a lower value of θ of 0.38, implying that states with loose fiscal rules have a lower propensity to spend cash on hand than states with stricter fiscal rule, for whom the value is 0.61 in column (3). The magnitude of target cash-on-hand is similar across these subsamples. The behavioral implication of these estimates will become clear after we see the estimates impatience and the risk aversion parameter which is inferred from the estimates of θ and target cash-on-hand.

5 SMM Estimation

5.1 Calibration

We estimate β and ρ as described later after calibrating the remaining parameters of the model.

[21] Craig et al. (2016) find a lower value of 0.25 for unemployment insurance system while JPP find a low value of around 0.1 for Italian consumers who report their target wealth.

5.1.1 Interest Rate

We compute the interest rate in each period for each state by dividing the beginning of period debt by the interest on debt averaged over the previous two year period. We set the interest rate in the model, r, to the overall average interest rate which is 5.5%. Since in our sample, each period is equal to two financial years, this is equivalent to an annual interest rate of 2.4%.

5.1.2 Income

We set the growth rate of permanent income in the model to the overall average computed from the data which is 1.5%, giving $G = 1.015$. The standard deviation of permanent shocks is computed as the standard deviation of permanent income, giving $\sigma_\psi = 0.05$. Transitory income is computed as the difference between log income and log permanent income from and set σ_ϵ to its standard deviation, giving $\sigma_\epsilon = 0.017$. We set the probability of zero income to 0.1% (ie. $\omega = 0.001$) as in Craig et al. (2016).[22]

Outside of those that we estimate, the only parameters that we calibrate are the real interest rate, r, and We set the probability of zero income $\omega = 0.001$ as in Craig et al. (2016).

[22]An alternative way to calibrate the probability of zero income is to compute, for each state, the ratio of income in period t to their average income over the sample period, $Y_{it}/\overline{Y_{i\cdot}}$, and set ω to the fraction of observations in which $Y_{it}/\overline{Y_{i\cdot}}$ is less than some cutoff, κ, as in Carroll (1992), who uses the Panel Study of Income Dynamics (PSID). Carroll (1992) sets κ to 0.1 by plotting the distribution of $Y_{it}/\overline{Y_{i\cdot}}$ and noting that there is some concentration of observations when $Y_{it}/\overline{Y_{i\cdot}} < 0.1$. He finds that the fraction of observations with $Y_{it}/\overline{Y_{i\cdot}} < 0.1$ is between 0.003 and 0.006 for various subsamples of the PSID and sets $\omega = 0.005$.

5.2 SMM Estimation and Standard Errors

The parameters of interest are the discount factor β and the coefficient of relative risk aversion ρ. Together, these two parameters govern consumption and savings behavior of states. We estimate for these parameters using Simulated Method of Moments where the moments we match are the target cash-on-hand across states and the covariance ratio. The target cash-on-hand for a state in the data is computed as the average cash to permanent income ratio for that state over time. The goal of SMM is to obtain parameter values that minimize the distance between the moments computed from the data and moments computed from model-simulated data, given a weighting matrix. Let m^d be a vector of moments computed from the data and let m^s be a vector of moments computed from data set simulated from the model for a given β and ρ. The SMM estimator is defined as

$$\hat{\beta} = argmin((m^d - m^s(\beta))'W(m^d - m^s(\beta)))$$

where W is a weighting matrix. The moments we match are the covariance ratio and target cash-on-hand.[23] For given β and ρ, we simulate a data set with 490 agents over 100 periods.[24] Because we have the same number of moments to match as parameters to estimate, the distance between the actual and simulated moment will go to zero and the choice of weighting matrix is immaterial for the solution, so we use an identity matrix.

We obtain standard errors using a parametric bootstrap. The estimated parameters

[23]We use MATLAB's fminsearch optimizer, restricting β and ρ to be positive.

[24]We simulate the agents with zero initial wealth for 200 periods then keep the last 100 periods. Doubling the size of the simulated data set we use for estimation produces roughly the same estimates.

inherit uncertainty from the distribution of the estimated θ value and the calculated target cash-on-hand. We assume a normal distribution for these parameters and we infer about the distribution of the parameter estimates as follows: we estimate a value of θ for each state and we compute the average cash-on-hand as an estimate of target cash-on-hand for each state. Having 49 of the parameters, we can compute the covariance matrix, V, of the state-specific target cash-on-hand and covariance ratios. This approach allows for stochastic variation due to differences across states as well as estimation uncertainty and gives larger values for the estimation uncertainty than asymptotic values calculated from a pooled regression of θ and average cash-on-hand. (Calculating the average is equivalent to regressing on a constant.) To implement the bootstrap, we draw 1000 pairs of θ and target cash-on-hand from $N(m^d, V)$ where N is a multivariate normal distribution.[25] For each draw we estimate the parameters of the model by SMM and record the estimates, matching the empirical moments to each of the 1000 pairs drawn. The standard error of the estimates is taken as the standard deviation of the estimates across the draws.[26]

Table 7 presents the estimates of β and ρ with the pooled sample in the first column and the sample of states with loose and tight fiscal rules in the second and third columns. In column (1), the discount factor β is found to be 0.90 which implies a discount factor for the state government of about 10% which is larger than the calculated interest rate of

[25] We redraw pairs in which the restriction

$$\theta > 1 - \frac{1+g}{(1+r)e^{\sigma_\psi^2}}$$

is not satisfied. This theoretical restriction is derived in Jappelli, Padula and Pistaferri (2008).

[26] Out of 1000 runs, 38 returned an error. When obtaining the standard errors for subsamples, around 200 runs returned an error.

5.5% implying impatience, consistent with target buffer stock savings.[27]. The relative risk aversion parameter ρ is 3.01—a moderate level consistent with typical findings in consumer models.

In column (2), we find a discount rate of 0.98, implying a discount rate of 2% for states with loose fiscal rules while, in column (3), we find a discount rate of 0.77, implying a much higher discount rate of about 23% for states with tight fiscal rules. States with loose fiscal rules have a coefficient of risk aversion of 1.78, lower than that of 3.83 for states with tight fiscal rules. The lower patience parameter for state with loose fiscal rules is reflected in the estimated value of θ being lower because, everything else equal, a more patient agent will consume a smaller fraction of cash on hand. States choose their own fiscal rules so the evidence here is not causal, in fact, the pattern seems to indicate that states with impatient politicians feel the need to impose stricter rules. While our evidence on this point is tentative, an avenue for future research may be the exploration of how fiscal institutional features affect state government fiscal behavior and modeling governments are buffer stock agents may be a fruitful framework for this.

6 Conclusion

This paper examines how U.S. states smooth expenditures over the business cycle. The question is motivated by two competing ideas. First, countries around the world are seeking to impose greater budgetary discipline on governments and are increasingly imposing budget

[27]Estimates of the discount factor and CRRA from consumer data are generally found to be between 0.92 and 0.96 and between 2.5 and 4, respectively (see, Cagetti (2003) and Gourinchas and Parker (2002)).

constraints (Lledó et al., 2017). Second, U.S. states are creating Rainy Day Funds as explicit mechanisms to use for inter-temporal smoothing.

The results in our paper bring two interesting pieces of information to the institutional structure question. First, states have substantial leeway in using cash and securities (cash) balances outside of rainy day funds, even if we exclude the earmarked pension and insurance trust funds. The cash balances are much larger, at least to the current time, than are the rainy day funds. Second, we find that state governments are about as patient as typical consumers and in addition behave with considerable risk aversion, also at the order of magnitude found for consumers.

These two findings together suggest that state governments within the institutional setup of our sample period behave as rational forward-looking patient risk averse agents. This likely reflects the institutional framework within which state government politicians operate and this framework may serve as model for other federations or even currency unions.

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7 Appendix

7.1 Other Data

In addition to the breakdown of government finances, we also have population data for each state, Consumer Price Indices, personal income and the years when Rainy Day Funds (and their predecessors) have first balance. Population data is from the Census Bureau where non-census year estimates are adjusted by the average growth rate between census years. CPI data for all items in the United States is obtained from FRED. The data on personal income is taken from the Bureau of Economic Analysis. The years when Rainy Day Funds (and their predecessors) have first balance are taken from the fiscal survey of states provided by the National Association of State Budget Officers (NASBO).

7.2 Numerical Solution

The model is solved using the method of endogenous grid points developed by Carroll (2006). To initialize the algorithm, we set an exogenous grid of end of period assets, a, and construct a fixed grid of cash-on-hand, coh. Our initial guess of the consumption function is $c' = coh$ which is consumption in the final period of life for finitely lived consumers. For each value of coh, we invert the Euler equation to obtain consumption in the current period:

$$\hat{c} = u'^{-1}(\beta(1+r)\mathbb{E}u'(c')) \tag{15}$$

where we drop the time subscripts. We then recover the value of cash-on-hand associated with $\hat{c}$ using the relationship $\widehat{coh} = \hat{c} + a$. This produces the endogenous grid of cash-on-hand $\widehat{coh}$ and gives us the pair $\{\widehat{coh}, \widehat{c}\}$ which we can use for interpolation and obtain a numerical approximation of the consumption function. We update our guess with $\hat{c}$ and repeat this procedure until the consumption function converges.[28]

We use a triple exponential grid for end-of-period assets with 200 grid points. For evaluating expectations, we discretize the distributions of the permanent and transitory shocks using Gauss-Hermite quadrature, each with 10 nodes (the zero income shock is the 11^{th} point for the transitory shocks).

7.3 Tables

Table 1: Summary Statistics I

	Mean	Std1	Std2
Total Revenue	11,043	4,821	2,466
Current Expenditure	9,349	3,249	2,276
Other Funds	2,574	7,303	989
Own Revenue	8,435	4,247	1,835
Total Expenditure	10,201	3,634	2,392
Cash and Security Holdings	10,335	10,253	3,989
Observations	980	980	980

Note: This table displays the summary statistics for the raw per capita data from 1974 to 2014 in 2015 dollars. Std1 denotes the cross sectional average of the standard deviation over time which is $\left[(1/T)\sum_t \left(X_{it} - \overline{X_i}\right)^2\right]^{\frac{1}{2}}$. $\overline{X_i}$ is the time average of X_{it} for states i and T is the number of years in the sample. Std2 denotes the time average of the standard deviation across states which is $\left[(1/n)\sum_i \left(X_{it} - \overline{X_t}\right)^2\right]^{\frac{1}{2}}$ for any variable X. $\overline{X_t}$ is the period t average of X_{it} across states. n is the number of states.

[28] We interpolate linearly and extrapolate for points outside of the grid.

Table 2: Summary Statistics II

	Top three components of Other Funds
California	Budget Stabilization Account, Special Fund for Economic Uncertainties, Safety Net Reserve
New York	SUNY University Operations, Infrastructure Investment Fund, Debt Service
Pennsylvania	Public Transport Trust fund, Public Transportation Assistance Fund, Race Horse Development Fund

Table 3: Summary Statistics III

	Corr (Cash, Tax)	Max (Cash)	Min (Cash)	Max (RDF)	Min (RDF)
California	0.74	1669	449	352	-178
Delaware	0.95	4898	1182	228	161
Hawaii	0.86	3967	1284	61	0
Indiana	0.94	2574	527	122	0
Kentucky	0.82	1891	467	95	0
Maryland	0.70	1661	323	173	0
Minnesota	0.51	2428	988	560	0
Montana	0.79	4231	1533	0	0
New Hampshire	0.76	953	82	167	0
New York	0.74	1539	225	69	0
Ohio	0.91	1648	390	122	0
Pennsylvania	0.97	2195	316	123	0
South Dakota	0.85	2743	940	258	0
Utah	0.90	1706	427	172	11
Washington	0.80	1553	456	176	0
Wyoming	0.84	28522	3565	1370	8

Note: This table reports the summary statistic for 16 states. We select the 3^{rd}, 6^{th}, 9^{th}.... 48^{th} state after arranging them alphabetically. All variables are in per capita 2015 US dollars term. The first column shows the correlation between cash and taxes. The second and third column report the range of the cash balances for the period 1975 - 2012. The fourth and fifth column report the range of the rainy day funds for the same period. In the beginning of the fiscal year 2010-11, California had a large fiscal deficit of $19 Billion. This was due to a 3.9% contraction of the economy in the preceding year. Later that year, it went into a full blown fiscal crisis with fiscal deficit going to $26 Billion. In this situation, the RDF temporarily had a negative balance. Source: https://www.nytimes.com/2010/11/18/us/18calif.html

Table 4: Summary Statistics of Buffer Stock Model Variables

	Means	Std1	Std2
Cash-on-hand/Permanent income	1.25	0.21	0.06
Consumption/Permanent income	1.31	0.24	0.07
Observations	882	882	882

Note: this table displays the summary statistics for the model data. We treat two years as one period. For flow variables, we take the sum of the two years. For stock variables we take the end-of-year value prior to the two-year period. The first period in the sample is 1975–1976. Permanent income is computed as a three year moving average. The first period used for estimation is 1976–1977. Std1 denotes the cross sectional average of the standard deviation over time which is $\left[(1/T)\sum_t \left(X_{it} - \overline{X_i}\right)^2\right]^{\frac{1}{2}}$. $\overline{X_i}$ is the time average of X_{it} for states i and T is the number of years in the sample. Std2 denotes the time average of the standard deviation across states which is $\left[(1/n)\sum_i \left(X_{it} - \overline{X_t}\right)^2\right]^{\frac{1}{2}}$ for any variable X. $\overline{X_t}$ is the period t average of X_{it} across states. n is the number of states. All variables are normalized by permanent income.

Table 5: **Calibration**

Parameter	Variable	Value	Sources
G	Growth Factor of P_t	1.027	Data
σ_ν	Std. Dev. of Trans. Shocks	0.017	Data
σ_ψ	Std. Dev. of Perm. Shocks	0.05	Data
r	Interest Rate	5.35%	Data
ω	Prob. of zero income	0.10%	Craig et al. (2014)

Table 6: Covariance ratio estimates for 1976-2012

	Whole sample	Loose Fiscal Rules	Strict Fiscal Rules
Cash-on-Hand (θ)	0.59***	0.39**	0.65***
	(0.06)	(0.18)	(0.03)
Target Cash-on-Hand	1.21***	1.18***	1.24***
	(0.04)	(0.04)	(0.07)
State Fixed Effects	Yes	Yes	Yes
N	931	456	475

*** 1% level of significance.
** 5% level of significance.

Note: this table presents the estimates of the covariance ratio defined in Jappelli, Padula and Pistaferri (2008). All variables are normalized by permanent income. Alaska is dropped from the sample in all specifications. The dependent variable is current expenditure of the state governments net of unexplained changes in taxes and medicaid. Cash-on-hand (coh) is sum of other funds and predicted taxes. ACIR index is an indicator of fiscal rules stringency constructed by the Advisory Commission on Intergovernmental Relations (1987). Strict fiscal rules correspond to states with ACIR index = 10 while loose fiscal rules correspond to states with ACIR index < 10. Flow variables are taken as two year sums while stock variables are taken as one year lags. Target cash-on-hand is the mean cash-on-hand over all periods for each state. We report the overall mean and the standard deviation across states of state-specific mean cash-on-hand. Robust standard errors, clustered by state, are in parenthesis.

Table 7: SMM Estimation

Parameter	Whole sample	Loose Fiscal Rules	Strict Fiscal Rules
β	0.90***	0.98***	0.77***
	(0.08)	(0.08)	(0.13)
ρ	3.01***	1.63	3.76***
	(0.58)	(1.04)	(1.06)

*** 1% level of significance.
** 5% level of significance.
* 10% level of significance.

Note: This table displays the estimated parameters and the model fit. The moments targeted are the average target cash-on-hand and the covariance ratio defined in Jappelli, Padula and Pistaferri (2008). Target cash-on-hand for a given state, coh_i^*, is defined as the average cash-on-hand over time. For estimation, we target the average of these across states. In the model, this is computed as the average cash-on-hand over time and across the simulated agents. The covariance ratio is $\theta = \frac{cov(coh_{it} - coh_i^*, c_{it})}{cov(coh_{it} - coh_i^*, coh_{it})}$ which is obtained by estimating the JPP regression on panel data simulated from the model. Bootstrap standard errors in parenthesis.

7.4 Figures

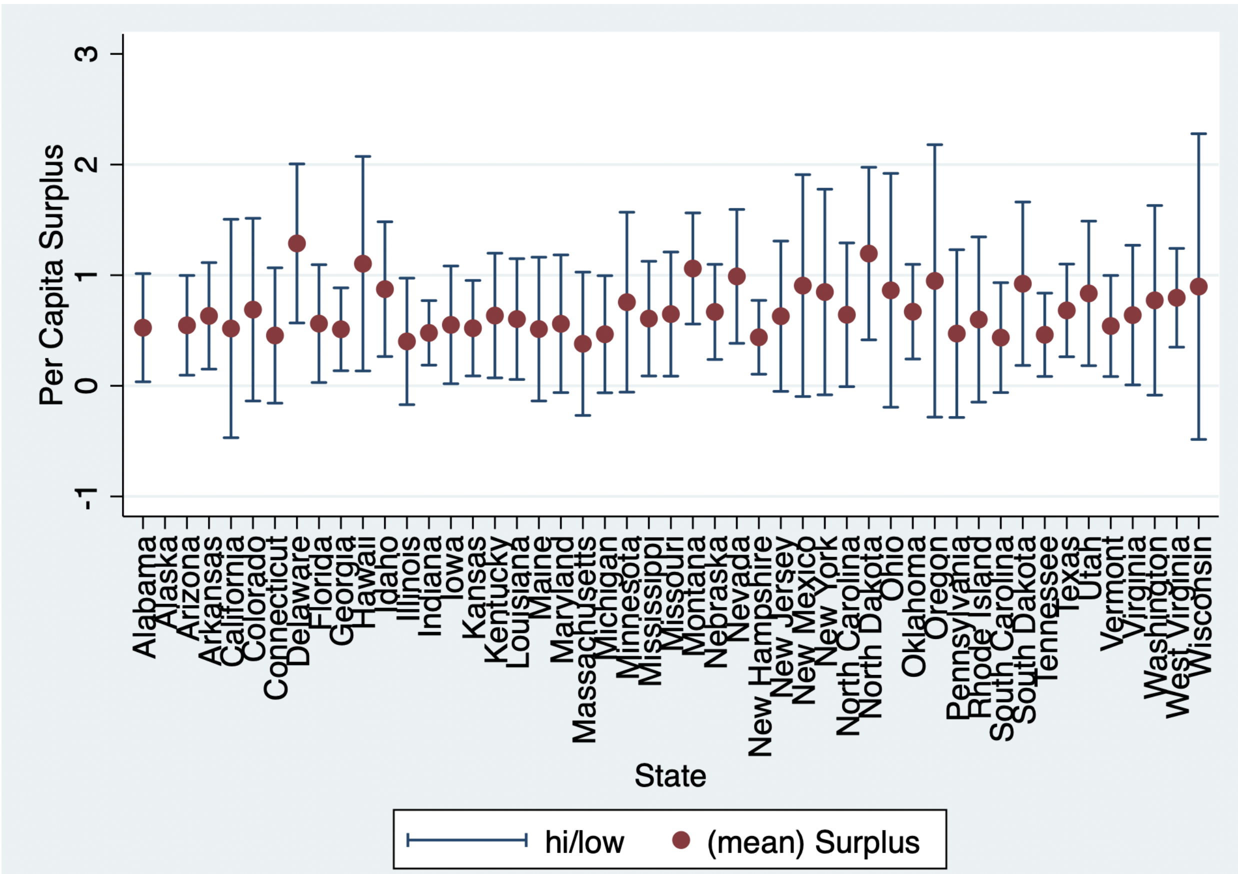

Figure 1: The red dots indicate the mean of surpluses in thousands of dollars per capita (2015 dollars) during the period 1975-2016. The blue bars are one standard deviations above and below the mean. Surplus = (Total Revenue - Current Expenditure)/State Population. The values for Alaska and Wyoming were 6 and 2.3 which are much higher than the rest of 48 states and hence not reported in the figure.

Figure 2: This figure shows changes in per capita current expenditure, taxes, cash and rainy day funds over two years averaged across states. Cash is a part of state's cash and security holdings which is not earmarked for any specific purpose. Top left panel plots cash and current expenditure(net of medicaid). Top right panel plots cash and taxes. Bottom left panel plots cash and rainy day funds. All variables are normalized in 2015 dollars.

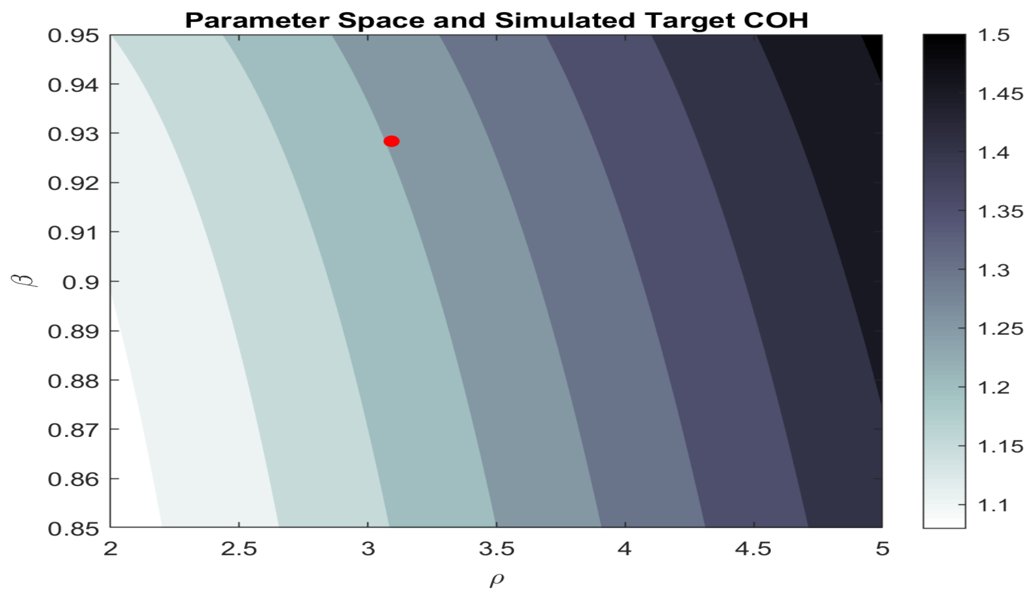

Figure 3: This figure plots the simulated target cash-on-hand for different values of ρ and β, fixing the income shocks in each simulation and using the same calibration as in the SMM estimation. For a given combination of ρ and β, we simulate a panel of 490 agents over 100 periods. The target cash-on-hand of a consumer is their average cash-on-hand over time. We compute each consumer's target cash-on-hand and take the average across consumers. We plot this average across consumers for each pair of ρ and β. Darker colors indicate higher target cash-on-hand. The red dot indicates the simulated target cash-on-hand given our SMM estimates of ρ and β.

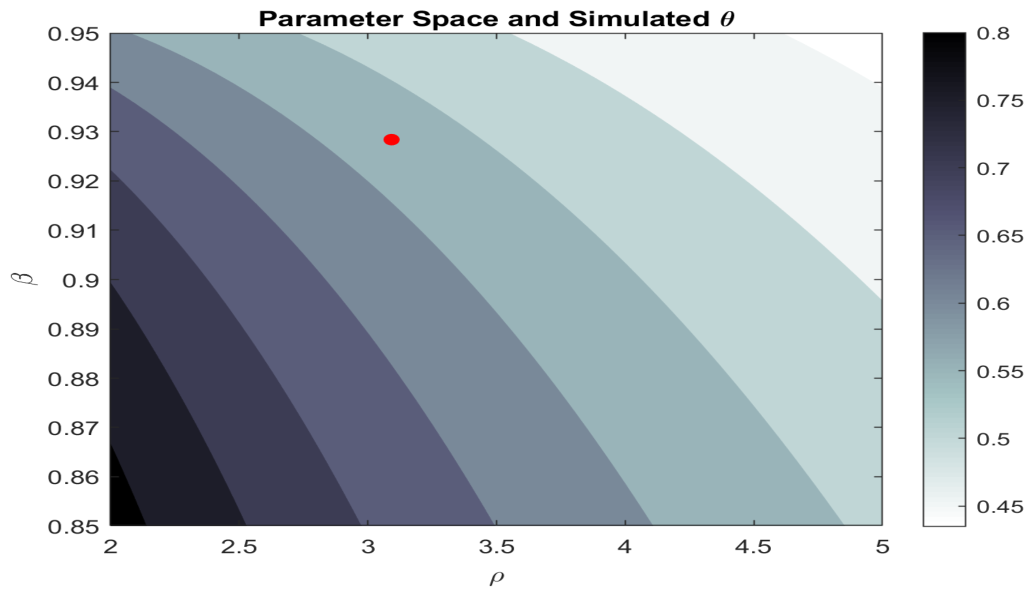

Figure 4: This figure plots the simulated covariance ratio for different values of ρ and β, fixing the income shocks in each simulation and using the same calibration as in the SMM estimation. For a given combination of ρ and β, we simulate a panel of 490 agents over 100 periods. We compute and plot the covariance ratio as in Jappelli, Padula and Pistaferri (2008) for each pair of ρ and β. Darker colors indicate higher covariance ratios. The red dot indicates the simulated covariance ratio given our SMM estimates of ρ and β.